Exercise 51

Find the given derivative by finding the first few derivatives and observing the pattern that occurs.

$$\frac{d^{99}}{dx^{99}}(\sin x)$$

Solution

Start calculating derivatives.

$$\frac{d^1}{dx^1}(\sin x) = \cos x$$

$$\frac{d^2}{dx^2}(\sin x) = -\sin x$$

$$\frac{d^3}{dx^3}(\sin x) = -\cos x$$

$$\frac{d^4}{dx^4}(\sin x) = \sin x$$

$$\frac{d^5}{dx^5}(\sin x) = \cos x$$

$$\frac{d^6}{dx^6}(\sin x) = -\sin x$$

$$\frac{d^7}{dx^7}(\sin x) = -\cos x$$

$$\frac{d^8}{dx^8}(\sin x) = \sin x$$

Notice that after every four derivatives, it goes back to $\sin x$, so we know that the 96th derivative is $\sin x$.

$$\frac{d^{96}}{dx^{96}}(\sin x) = \sin x$$

$$\frac{d^{97}}{dx^{97}}(\sin x) = \cos x$$

$$\frac{d^{98}}{dx^{98}}(\sin x) = -\sin x$$

$$\frac{d^{99}}{dx^{99}}(\sin x) = -\cos x$$